**Module 3: Trigonometric Identities, Inverse Functions, and Applications**

**III. Solution of Trigonometric Equations**

After completing this section, you should be able to:

* solve trigonometric equations

In *College Algebra,* you learned how to solve several types of equations. One thing you learned is that a quadratic equation has at most two real-valued solutions.

An equation involving a trigonometric expression, by contrast, may have infinitely many solutions.

**Example III.1:** Solve the equation sin2 *x* – 3 cos2 *x* + 2 = 0.

Solution:

|  |  |  |  |
| --- | --- | --- | --- |
| sin2 *x* – 3 cos2 *x* + 2 | = 0 | |  |
| sin2 *x* – 3(1 – sin2 *x*) + 2 | = 0 | | Apply the identity cos2 *x* = 1 – sin2 *x*. |
| sin2 *x* – 3 + 3 sin2 *x* + 2 | = 0 | | Multiply out. |
| 4 sin2 *x* – 1 | = 0 | | Simplify. |
| sin2 *x* | = 1/4 | | Solve for sin2 *x*. |
| sin *x* | = ± ½ | | Apply the principle of square roots to solve for sin *x*. |
| The equation sin *x* = ½ has solutions **π/6** in quadrant I and **5π/6** in quadrant II.    The equation sin *x* = –½ has solutions 7π/6 in quadrant III and 11π/6 in quadrant IV. | |  | |

Any angle coterminal with π/6, 5 π/6, 7π/6, or 11π/6 is also a solution. Coterminal angles differ by a multiple of 2π.

Conclusion: The solutions are π/6 + 2*k*π, 5π/6 + 2*k*π, 7π/6 + 2*k*π, and 11π/6+ 2*k*π, where *k* is any integer.

Often, the solutions of primary interest are those between 0 and 2π, or between 0° and 360°. Typically, the instructions specify the interval of interest, which indicates the system of measurement (radians or degrees).

**Example III.2:** Solve the equation cos 2*x* = cos *x* in the interval [0, 2π).

Solution:

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| cos 2*x* | = cos *x* |  |
| 2 cos2 *x* – 1 | = cos *x* | Apply a double-angle identity for cosine. |
| 2 cos2 *x* – cos *x* – 1 | = 0 | Subtract cos *x* from both sides. |
| (2 cos *x* + 1)(cos *x* – 1) | = 0 | Factor. |
|  |  |  |
| The principle of zero factors tells us that either | | |
|  | | |
| 2 cos *x* + 1 = 0 | or | cos *x* – 1 = 0 |
| 2 cos *x* = –1 |  | cos *x* = 1 |
| cos *x* = –1/2 |  |  |
|  |  |  |
| *x* = 2π/3 in quadrant II | or | *x* = 0 |
| or 4π/3 in quadrant III |  |  |

The solutions in the interval [0, 2π) are 0, 2π/3, and 4π/3.

**Example III.3:** Solve the equation 3 sin2 *t* – 2 sin *t* – 2 = 0 in the interval [0°, 360°).

Solution:

|  |  |
| --- | --- |
| 3 sin2 *t* – 2 sin *t* – 2 = 0 | The equation involves sin *t* and sin2 *t*. |
| Let *u* = sin *t*. |  |

Then 3*u*2 – 2*u* – 2 = 0. This is a quadratic equation that does not factor.

Apply the quadratic formula with *a* = 3, *b* = –2, and *c* = –2:

The discriminant is *b*2 – 4*ac* = (–2)2 – 4(3)(–2) = 4 + 24 = 28.

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|  |  |  |
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| sin *t* ≈ 1.2153  There is no solution because sine values cannot be larger than 1. | or | sin *t* ≈ –0.5486  There are solutions in quadrants III and IV, since the sine is negative.  Using a calculator in DEGREE mode, the reference angle having a sine of 0.5486 is found to be equal to 33.27°.    The quadrant III solution is 180° + 33.27° =213.27°, and the quadrant IV solution is 360° – 33.27° = 326.73°. |

The solutions in the interval [0°, 360°) are 213.27° and 326.73°.

**Example III.4:** Solve tan 3*t* = 0 in the interval [0, 2π).

Solution:

This problem is a little tricky because it involves 3*t* rather than *t*:

|  |  |
| --- | --- |
| 0 ≤ *t* < 2π | A solution *t* is in the interval [0, 2π). |
| 0 ≤ 3*t* < 6π | Multiply the inequality by 3. |

A solution of the form 3*t* is in the interval [0, 6π).

Let *u* = 3*t*.  
Now solve the equation tan *u* = 0 in the interval [0, 6π).  
The solutions are *u* = 0, π, 2π, 3π, 4π, and 5π.  
Substituting for *u*, the solutions are 3*t* = 0, π, 2π, 3π, 4π, and 5π.  
Dividing by 3 to solve for *t*, the solutions are *t* = 0, π/3, 2π/3, π, 4π/3, and 5π/3.

**Example III.5:** Solve 2 sin *x* – cos *x* = 1 in the interval [0, 2π).

Solution:

|  |  |  |
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| 2 sin *x* – cos *x* | = 1 |  |
| 4 sin2 *x* – 4 sin *x* cos *x* + cos2 *x* | = 1 | Square both sides. |
| 3 sin2 *x* + (sin2 *x* + cos2 *x*) – 4 sin *x* cos *x* | = 1 | Rearrange to group sin2 *x* + cos2 *x*. |
| 3 sin2 *x* + 1 – 4 sin *x* cos *x* | = 1 | Apply the identity sin2 *x* + cos2 *x* = 1. |
| 3 sin2 *x* – 4 sin *x* cos *x* | = 0 | Subtract 1 from both sides. |
| (sin *x*)(3 sin *x* – 4 cos *x*) | = 0 | Factor. |
| sin *x* = 0 or 3 sin *x* – 4 cos *x* | = 0 | Apply the principle of zero products. |

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| sin *x* = 0  *x* = 0 or  *x* = π | or | 3 sin *x* – 4 cos *x* = 0  3 sin *x* = 4 cos *x*      This equation has a solution in quadrant I and in quadrant III. |  |
| Using a calculator, determine:  in quadrant I,  ≈ 0.9273 radians, and  in quadrant III, ≈ 4.069 radians | |

Each potential solution must be checked in the original equation.

**Checks:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* = 0:  2 sin 0 – cos 0  ? 1 0 – 1 ? 1 –1 ≠ 1  *x* = 0 is not a solution. | |  |  |  | | --- | --- | --- | | : |  |  | |  | ? 1 |  | |  | ? 1 | Consult quadrant I triangle. | |  | = 1 | Simplify. | | ≈ 0.9273 is a solution. | | | |  |
| *x* = π:  2 sin π – cos π ? 1 0 – (–1) ? 1 1 = 1  *x* = π is a solution. | |  |  |  | | --- | --- | --- | |  |  |  | |  | ? 1 |  | |  | ? 1 | See reference triangle. | |  | ? 1 | Simplify. | | –1 | ≠ 1 |  | | ≈ 4.069 is not a solution. | | | |  |

The solutions in the interval [0, 2π) are π and  ≈ 0.9273 radian.

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